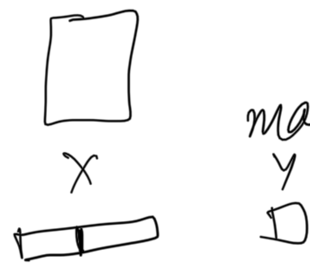
Probabilistic ML

$X \qquad Y$

$P(X, Y)$

benign

malignant



Tumor: color , size

color → dark red, pink

size → large, small

$P(\text{color} = \text{dark red}, \ \text{size} = \text{large}, \ \text{class} = \text{benign})$

$P($

What is the class?

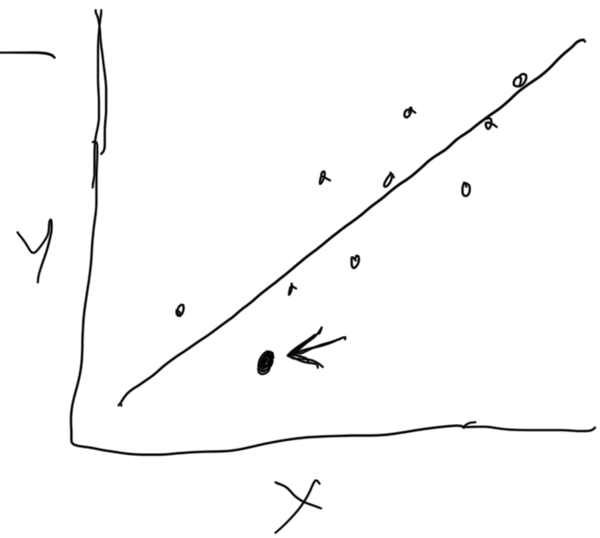
given (dark red, large)

$P(\text{class} = \text{benign} \mid \text{color} = \text{dark red}, \ \text{size} = \text{large})$

random variables

$X \rightarrow$ take any value in a domain

Monday March 29



X, Y

$X | Y$

$Y | X$

$$\boxed{X = \begin{cases} 1 & \text{if face is heads} \\ 0 & \text{if face is tails.} \end{cases}}$$

Probability Distribution

Probability Distribution

Discrete $\rightarrow$ pmf

probability mass function.

Continuous $X \in (-\infty, \infty)$

$X \in (0, \infty)$

$X \in (0, 1)$

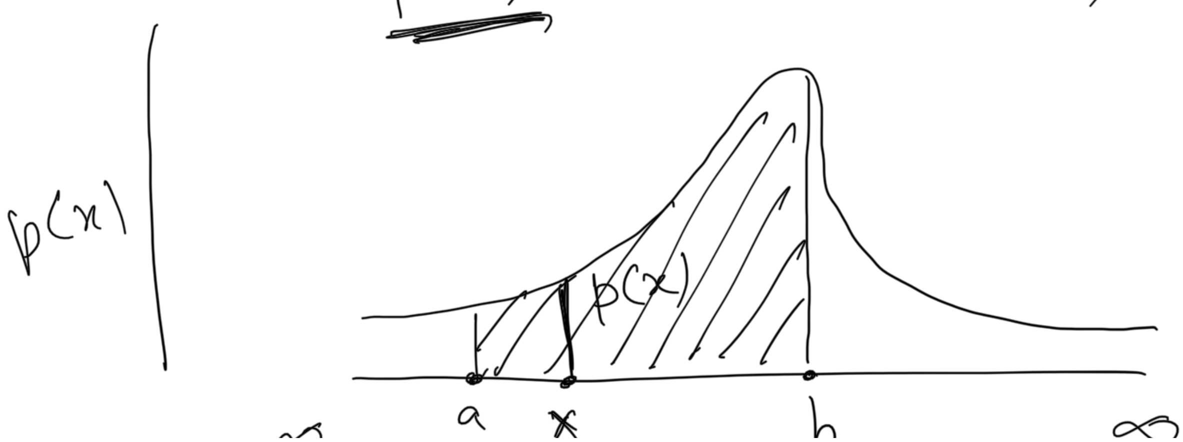
$P(X = x)$

$P(X = 0.25)$? $\rightarrow \simeq 0$

---

probability density function $p(\ )$

$p(x)$



$-\infty$

$$\boxed{\int_{-\infty}^{\infty} p(x)\,dx = 1}$$

$P(X = x)$ $\quad$ $P(x)$

probability

$P(0.3)$

$P(X = 0.3)$

$P_X(0.3)$

$P_Y(0.3)$

$p(x)$ $\quad$ $p(0.3)$ $\rightarrow$ $p_X(0.3)$, $p_Y(0.3)$

pdf

pmf.

$$X = \begin{cases} 1 & \text{if } q = \text{heads} \\ 0 & \text{if } q = \text{tail} \end{cases}$$

$$Y = \begin{cases} 1 & \text{if } p = \text{heads} \\ 0 & \text{if } p = \text{tails.} \end{cases}$$

$\boxed{X, Y}$

$P(X=1, Y=1)$

$P(X=1, Y=0)$

$X \mid Y=1$

Bayes Rule

$$\boxed{P(X=x \mid Y=y) = \frac{P(Y=y \mid X=x)\,P(X=x)}{\sum_{x' \in \mathcal{X}} P(X=x')\,P(Y=y \mid X=x')}}$$

What we really want:

$P(Y=1 \mid X=1)$

→ Probability that I have cancer, given that I have tested +ve.

$$P(y=1 \mid x=1) = \frac{P(y=1)\,P(x=1 \mid y=1)}{P(y=0)\,P(x=1 \mid y=0) + P(y=1)\,P(x=1 \mid y=1)}$$

0.004

0.8 0.8

$1 - P(y=1) = 0.996$

$\boxed{P(x=1 \mid y=0)}$ is different from

$\boxed{P(x=1 \mid y=1)}$

and $\boxed{P(x=1 \mid y=0) \neq 1 - P(x=1 \mid y=1)}$

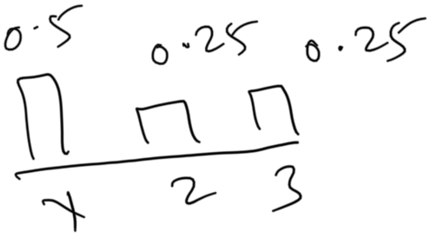
Assume : $P(x=1 \mid y=0) = \underline{0.1}$

$$P(y=1 \mid x=1) = \frac{0.004 * 0.8}{0.996 \times 0.1 + 0.004 * 0.8}$$

$$= \underline{0.031} \ll \underline{0.8}$$

Wed 03/31

X



$E[\ (\ )]$ $\qquad \sum g(x)\, P(X=x)$

$$\mathbb{E}[g(X)] = \sum_{x \in \mathcal{X}} g \ldots$$

<u>Ex:</u>

$$\text{dom}(X) = \{1, 2, 3\}$$

$$g(x) = \begin{cases} 7 & \text{if } x = 1 \\ 3 & \text{if } x = 2 \\ -1 & \text{if } x = 3 \end{cases}$$

$$\mathbb{E}[g(x)] = g(1)\,P(x=1) + g(2)\,P(x=2) + g(3)\,P(x=3)$$

$$= 7 * 0.5 + 3 * 0.25 + (-1) * 0.25$$

If X – continuous

$$\mathbb{E}[g(x)] = \int g(x)\,p(x)\,dx$$

$$\mathbb{E}[g(x)] = \int_{\mathcal{X}} g(x)\, p(x) \dots$$

$$\mathbb{E}[c] = \sum_{x \in \mathcal{X}} c\, P(X=x)$$

$$= c \sum_{x \in \mathcal{X}} P(X=x)$$

$$= c$$

Expectation of a random variable:

$$\mathbb{E}[X] = \sum_{x \in \mathcal{X}} x\, P(X=x)$$

$$\mathbb{E}[X] = \int x\, P(x)\, dx$$

Mean of X (μ) = $\mathbb{E}[X]$

$$\mathrm{var}[X] = \mathbb{E}\left[\underline{(X - \mathbb{E}[X])^2}\right]$$

$$= \mathbb{E}\left[X^2 + (\mathbb{E}[X])^2 - \underline{2}\, X\, \underline{\mathbb{E}[X]}\right]$$

$$= \mathbb{E}[X^2] + (\mathbb{E}[X])^2 - 2\mathbb{E}[X]\mathbb{E}[X]$$

$$= \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

Binomial:

$X \equiv$ # of heads observed when a coin is tossed n times with a $\underbrace{\text{success probability}}_{\text{heads}}$ of θ. $(0 \le \theta \le 1)$

$$\boxed{P(X = k) = \binom{n}{k} \theta^k (1-\theta)^{n-k}}$$

$$X \sim \text{Bin}(n, \theta)$$

Bernoulli $\quad X \quad \text{dom}(X) = \{0, 1\}$

θ

---

if X is Binomial:

$$\mathbb{E}[X] = \sum_{x=0}^{n} x \binom{n}{x} \theta^x (1-\theta)^{n-x}$$

$= n\theta$

if $n = 20$, $\theta = 0.3$

$\mathbb{E}[X] = 6$

Poisson

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Cummulative Distribution fn

$$F(a) = P[0 < X \leq a]$$

$$= \int_0^a p(x)\,dx$$

Gaussian

X is cantinous

$$-\infty \leq X \leq \infty$$

domain $(-\infty, \infty)$

$\text{dom}(X) = (\quad , \quad)$

Central Limit Theorem

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right]$$

Multi-variate Gaussian

$$\underline{\mu} = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix} \qquad \Sigma = \begin{bmatrix} & \\ & \end{bmatrix}$$

$D \times 1$ $\qquad$ $D \times D$

$$\text{pdf}(X) = \mathcal{N}(X \mid \mu, \Sigma)$$

$$= \frac{1}{(2\pi)^{D/2} |\Sigma|^{D/2}} \exp\left[-\frac{1}{2} (x-\mu)^T \Sigma^{-1} (x-\mu)\right]$$

$1 \times D$ $\quad$ $D \times D$ $\quad$ $1 \times D$