Bayesian Classification

Fri April 9

What is the probability of x^* to be malignant?

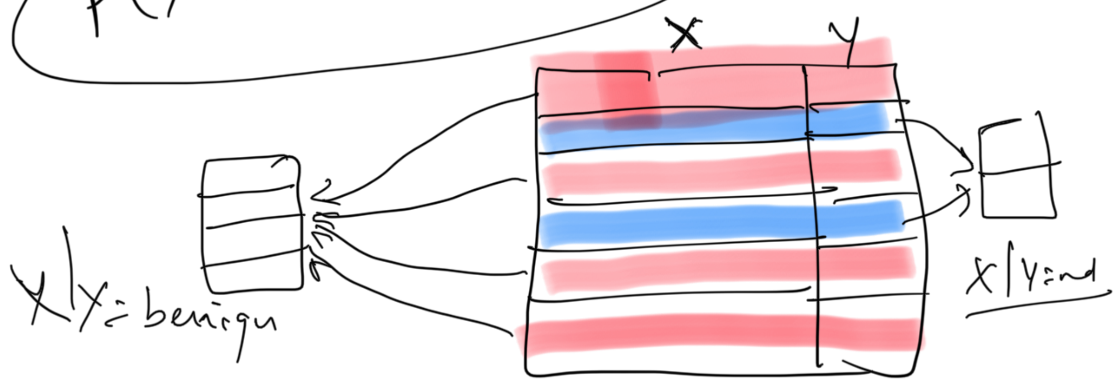
$\cancel{P(X=x^* \mid Y=\text{malignant})}$

$\cancel{P(Y=\text{malignant})}$

$P(Y=\text{malignant} \mid X=x^*)$

$P(X=x^* \mid Y=\text{malignant})$

$P(X=x^* \mid Y=\text{benign})$



Assume that X had only one binary feature

$X - \{0, 1\}$

$p(X=1) \leftarrow$

$p(X=0)$

$$\boxed{p(X=0) + p(X=1) = 1}$$

$X - \{0, 1, 2\}$

$p(X=0)$

$p(X=1)$ → 2 parameter.

$p(X=2)$

Assume X is described using 2 features

$X_1 \in \{0,1\}\ X_2 \in \{0,1\}$

$P(X_1, X_2)$ → 3 parameters

$$\begin{array}{c|c|cc} & & \multicolumn{2}{c}{X_2} \\ & & 0 & 1 \\ \hline X_1 & 0 & - & - \\ & 1 & - & - \end{array}$$

Assume X is described using D binary features.

$P(x_1\ x_2\ \cdots\cdots\ x_D) = \underline{2^D - 1}$

parameters.

$P(x_1\ x_2\ \cdots\ x_D \mid y = \text{benign})$

& $P(x_1\ x_2\ \cdots\ x_D \mid y = \text{malignant})$

For Bayesian classifier $\longrightarrow$ $\underline{2(2^D - 1)}$

# of classes.

$2\left(\prod_{i=1}^{D} D_i - 1\right)$ $D_i \rightarrow$ # possible values for the i^{th} features.

$D = 10 \Rightarrow 2 * (2^{10} - 1)$ parameters

$\underline{2046}$

$\underline{P(X_1 = x_1, X_2 = x_2, \cdots\cdots\cdots X_D = x_D)}$

$= \underline{P(X_1 = x_1)\ P(X_2 = x_2) \cdots\ P(X_D = x_D)}$

$\rightarrow$ Only true if $X_1\ X_2\ \cdots X_D$ are independent of each other.

independent ...

→ Only D parameters.

For Bayesian Classification, we only need 2D parameters

E.x $D = 10$: 20 parameters.

$P(Y = \text{malignant} \mid X = x^*)$

$$= \frac{P(X = x^* \mid Y = \text{malig.})\, P(Y = \text{malig.})}{P(X = x^* \mid Y = \text{malig.})\, P(Y = \text{malig.}) + P(X = x^* \mid Y = \text{ben.})\, P(Y = \text{ben.})}$$

θ : $P(Y = \text{malig.})$

$1 - \theta$: $P(Y = \text{benign})$

θ_{benign} ← vector of length D

→ $\theta_{1\,\text{benign}}, \theta_{2\,\text{beny.}}, \ldots\ldots$

$\theta_{\text{malig.}}$ ← vector of length D

Y - m or b.

$N = 10$

$D = 3$

Naive Bayes Classification (MLE)

Training

y

$$\theta_{benign} = \frac{5}{10} = 0.5$$

$$\theta_{malig} = \frac{5}{10} = 0.5$$

$x \mid y \quad x_1 \; x_2 \; x_3 \mid y$

$P(X_1 = cir \mid y = b) = \frac{2}{5}$	$P(X_1 = cir \mid y = m) = \frac{3}{5}$
$P(X_1 = ov \mid y = b) = \frac{3}{5}$	$P(X_1 = ov \mid y = m) = \frac{2}{5}$
$P(X_2 = lg \mid y = b) = \frac{2}{5}$	$P(X_2 = lg \mid y = m) = \frac{4}{5}$
$P(X_2 = sm \mid y = b) = \frac{3}{5}$	$P(X_2 = sm \mid y = m) = \frac{1}{5}$
$P(X_3 = ligh \mid y = b) = \frac{3}{5}$	$P(X_3 = li \mid y = m) = \frac{2}{5}$
$P(X_3 = dk \mid y = b) = \frac{2}{5}$	$P(X_3 = dk \mid y = m) = \frac{3}{5}$

$P(Y = \cancel{mal}\ benign \mid X = \underline{cir, sm, li})$

$$= \frac{P(X^t = cir, sm, li \mid y = benign)\ P(y = benign)}{Z}$$

$$= \frac{\frac{2}{5} * \frac{3}{5} * \frac{3}{5} * \frac{1}{2}}{\frac{2}{5} * \frac{3}{5} * \frac{3}{5} * \frac{1}{2} + \frac{3}{5} * \frac{1}{5} * \frac{2}{5} * \frac{1}{2}}$$

$$= \frac{18}{18+6} = \frac{3}{4} = 0.75$$

$$P(y = malig \mid X = \ldots) = 1 - \frac{3}{4} = 0.25$$